Elettra Sincrotrone Trieste

An analytical model for beam loading studies at Elettra

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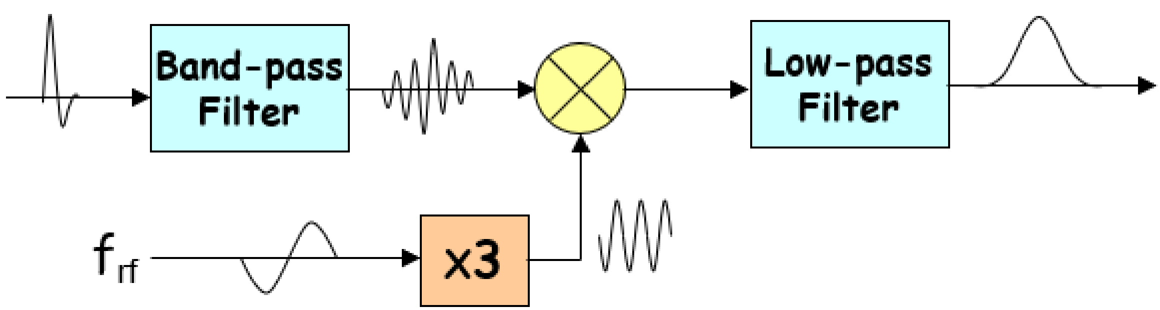
Motivations

- All synchrotrons operate with a dark gap to avoid ion instabilities
- A gap produces transient beam loading on the beam due to harmonic cavities
- Changes of longitudinal bunch position degrade the effect of harmonic cavities and generate troubles in any electronic device interacting with the beam and synchronous with the RF clock

Ex. multi-bunch feedback front-end



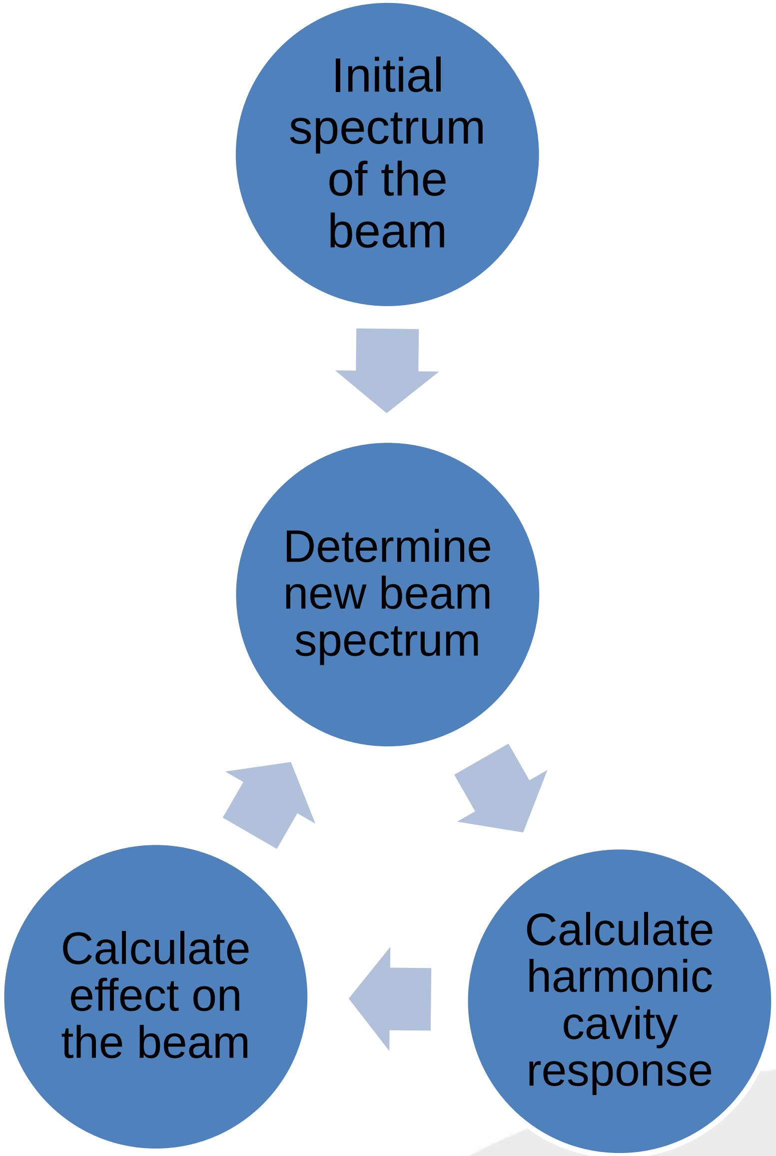
For Elettra 2.0 we need an estimation of the transient beam loading due to the third harmonic cavity to evaluate impacts on MBF systems (longitudinal modulator working at the 4th RF harmonics), crab cavities, hybrid mode, …

Simulator structure

Simulator developed in MATLAB

Analytical frequency-domain model of the beam and harmonic cavity

- The beam is represented by a periodic (ω_r) signal made of pulses at ω_{RF} modulated in amplitude and phase; has a given spectrum
- The harmonic cavity is a dynamic system with an INPUT (beam) and an OUTPUT (harmonic voltage)
- The harmonic gap voltage modifies the bunch length and position (long. displacement of the bunches with respect to synchronous phase)
- Iterate process until a stable, consistent solution is found (normally a few iterations are sufficient to converge)



Harmonic cavity model and transfer function

$$V_h = 2R_s I_b \cos\psi$$

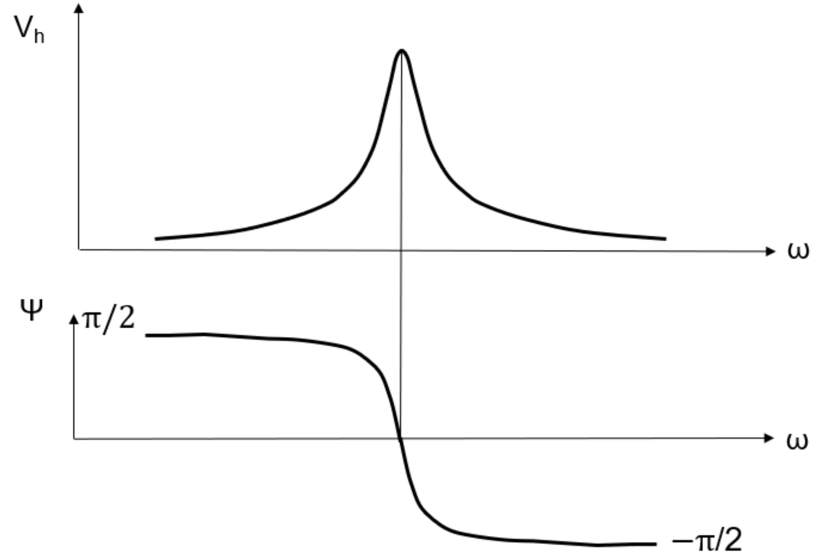
$$\tan\psi = \frac{2Q\delta\omega}{\omega_h}$$

$$\delta\omega = \omega_h - \omega_{nRF}$$

n = harmonic of the RF

$$P = \frac{V_h}{2Rs} = V_h I_b \cos\psi$$

$$TF_h(\omega)\begin{cases} V_h(\omega) = I_b \dfrac{R_s}{Q}\dfrac{\omega_h}{\omega_h-\omega}\sin\tan^{-1}\left(\dfrac{2Q(\omega_h-\omega)}{\omega_h}\right) \\ \psi(\omega) = \tan^{-1}\left(\dfrac{2Q(\omega_h-\omega)}{\omega_h}\right)\end{cases}$$



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Harmonic cavity response

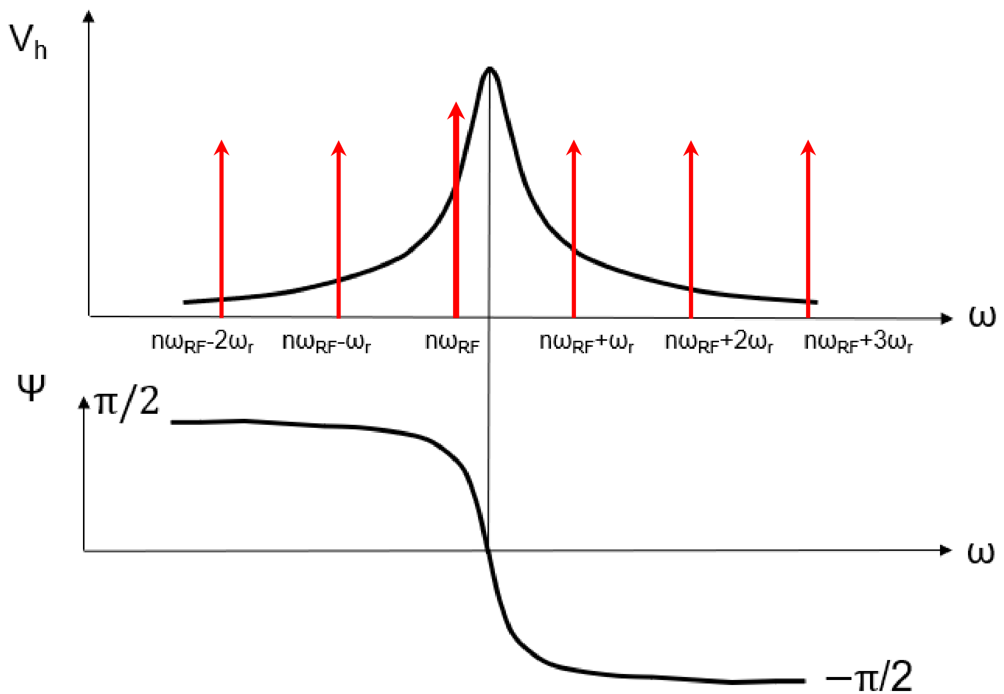
Bunch Form Factor

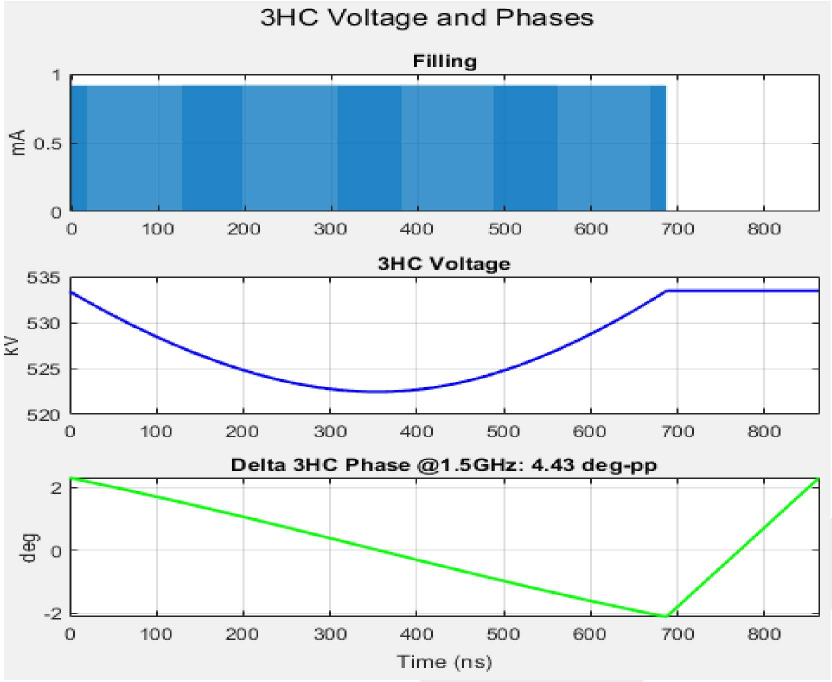
Beam signal seen by the harmonic cavity, periodic at ω_r

$$I_b(t) = e^{-\frac{(n\omega_{RF}\sigma)^2}{2}} \cdot BF(t) \cdot e^{j\Delta\varphi_s(t)} \cdot e^{jn\omega_{RF}t}$$

$BF(t)$ = filling function (array of *h* arbitrary values, *h* = number of buckets)

Harmonic cavity response: $$V_h(\omega) = \mathcal{F}(I_b)(\omega) \cdot TF_h(\omega)$$





Harmonic cavity effects on the beam - 1st method

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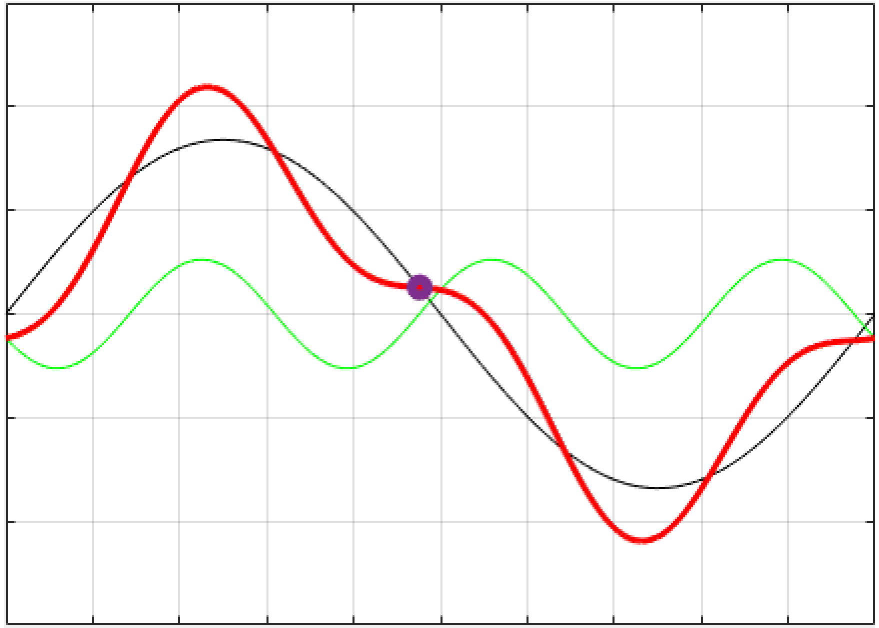
Total voltage seen by the beam



The total voltage seen by each bunch is constant and only depends on U_0

$$\Delta V_h = V_h \sin(\Delta\varphi_h + n\Delta\varphi_s)$$

$$\Delta V_{RF} = V_{RF} \sin(\varphi_s + \Delta\varphi_s) - V_{RF} \sin\varphi_s$$

Must be equal

Synchronous phase: $\varphi_s = \pi - \sin^{-1}\left(\frac{U_0}{eV_{RF}}\right)$

$\Delta\varphi_h = f(\Delta\varphi_s)$: $\Delta\varphi_h = \sin^{-1}\left(\frac{V_{RF}}{V_h} \sin(\varphi_s + \Delta\varphi_s) - \sin\varphi_s\right) - h\,\Delta\varphi_s$

Get the inverted function $\Delta\varphi_s = f(\Delta\varphi_h)$ by interpolation

Harmonic cavity effects on the beam - 2nd method

RF potential

$$U(\varphi) = -\frac{c\,\alpha}{EC\omega_{RF}}\int_0^{\varphi}(eV_{tot}(\varphi') - U_0)d\varphi'$$

$$V_{tot}(\varphi) = V_{RF}\sin(\varphi + \varphi_s) + V_h\sin(n\varphi + \varphi_h)$$

$$U(\varphi) = -\frac{c\,\alpha\, e}{EC\omega_{RF}}\left\{V_{RF}[\cos\varphi_s - \cos(\varphi + \varphi_s)] + \frac{V_h}{n}[\cos\varphi_h - \cos(n\varphi + \varphi_h)] - \frac{U_0\varphi}{e}\right\}$$

Charge distribution

$$\Psi(\varphi) = \overline{\Psi}\, e^{-\frac{U(\varphi)}{\alpha^2\sigma_e^2}}$$

$$\left(\overline{\Psi} = \frac{1}{\int_{-\pi}^{\pi} e^{-\frac{U(\varphi)}{\alpha^2\sigma_e^2}}d\varphi}\right)$$

Median value (bunch position change $\Delta\varphi_s$)

$$\overline{\varphi} = \frac{\int_{-\pi}^{\pi}\varphi\Psi(\varphi)\,d\varphi}{\int_{-\pi}^{\pi}\Psi(\varphi)\,d\varphi}$$

Bunch Length and Synchrotron Frequency

RMS bunch length

$$\sigma = \sqrt{\frac{\int_{-\pi}^{\pi}(\varphi - \overline{\varphi})^2 \Psi(\varphi)\, d\varphi}{\int_{-\pi}^{\pi}\Psi(\varphi)\, d\varphi}}$$

Synchrotron frequency

$$\omega_s = \omega_{RF}\sqrt{\frac{-\alpha V_{RF}\cos\varphi s}{2\pi E_0 h}}$$

$-\alpha V_{RF}\cos\varphi s \rightarrow \frac{dV_{tot}}{d\varphi}$

Simulator Implementation

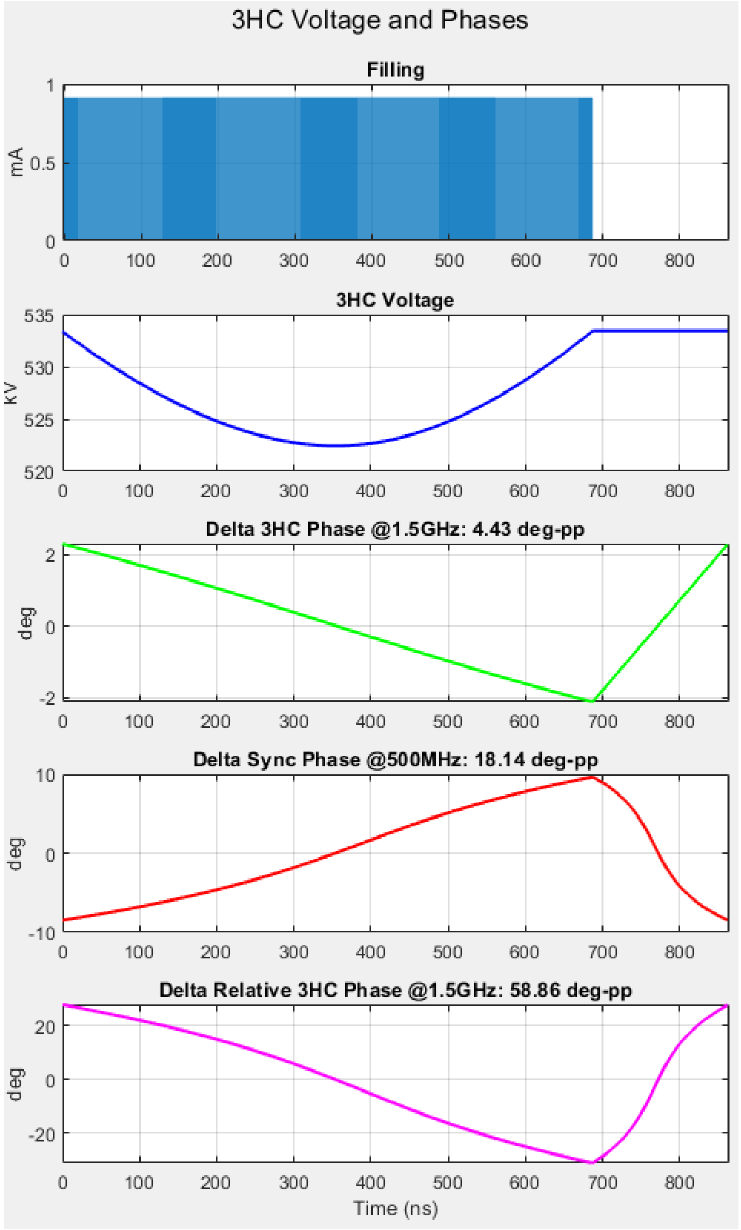
$[\varphi_s, V_h, \Delta\varphi_h, \Delta\varphi_s]$ = **Simulator3HC_function**(*data, DoPlot, UseFormFactor, UseAnalyticalModel*);

Data:

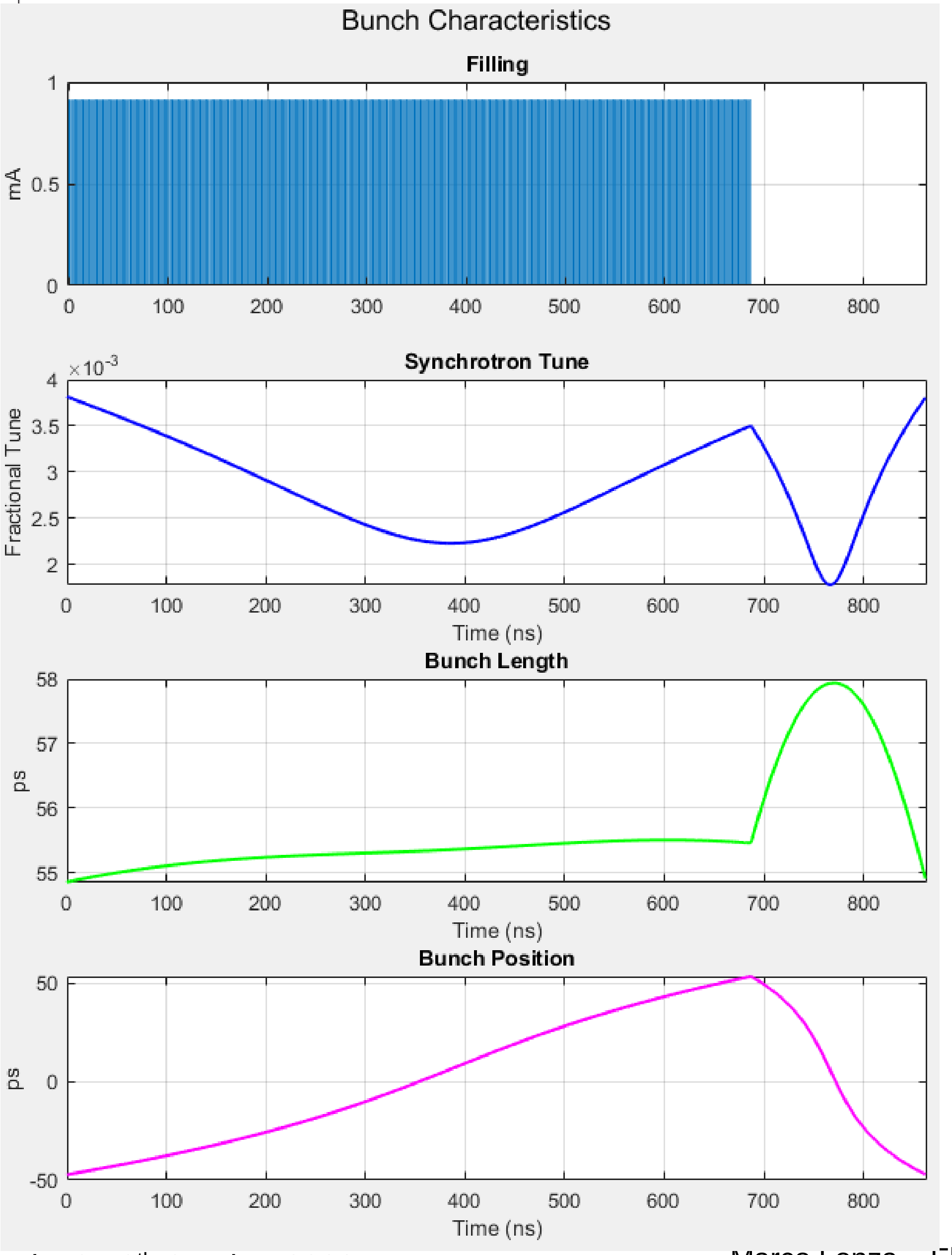
```
Ib = 0.315;              % beam current (A)
Q = 2.0e8;               % harmonic cavity quality factor
Rs = 18e9;               % harmonic cavity shunt impedance (Ohm)
fRF = 500e6;             % RF frequency (Hz)
h = 432;                 % harmonic number
FF = 0.80;               % filling factor
detuning = 70.0e3;       % detuning (Hz)
VRF = 1.68e6;            % RF peak voltage (V)
EnergyLoss = 256e3;      % energy loss (eV)
alpha = 1.6e-3;          % momentum compaction factor
E = 2.0e9;               % beam energy (eV)
c = 2.998e8;             % speed of light (m/s)
C = 259.2;               % ring circumference (m)
sigma_e = 0.9e-3;        % energy spread
```

Elettra superconductive passive Third Harmonic Cavity (3HC)

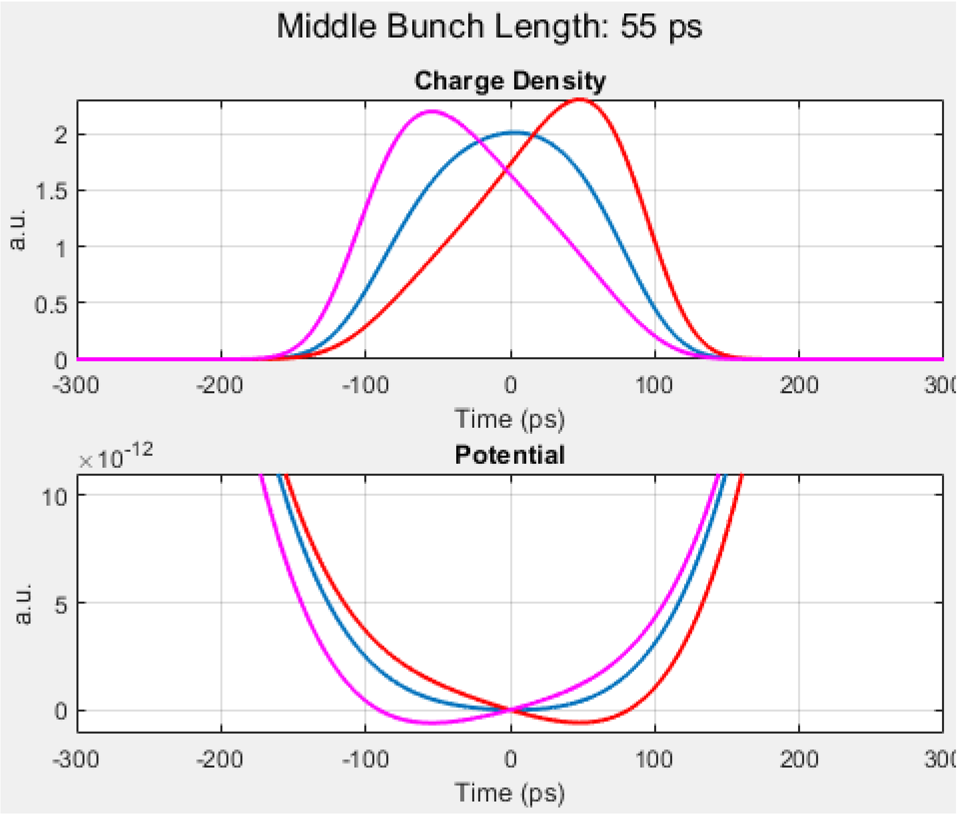
Simulator results: harmonic voltage and phase, beam loading



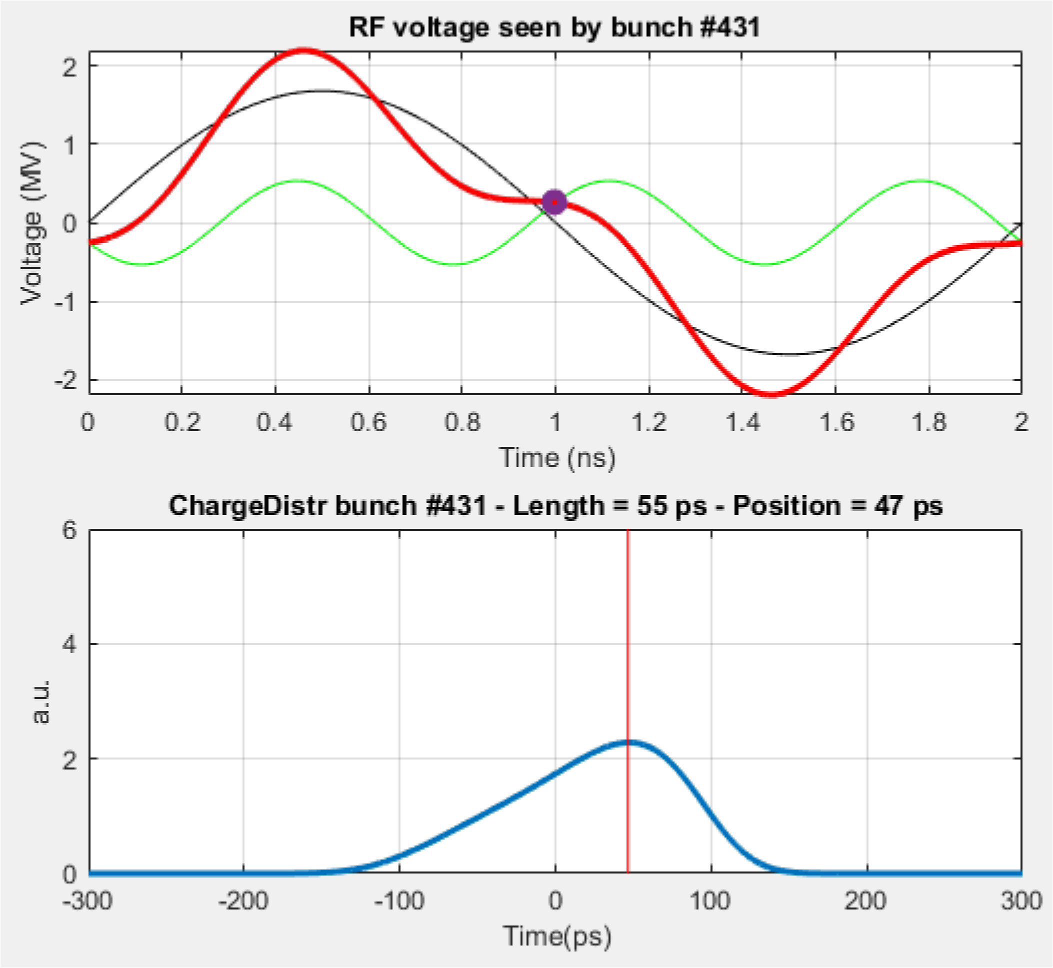
Simulator Results: bunches tune and length



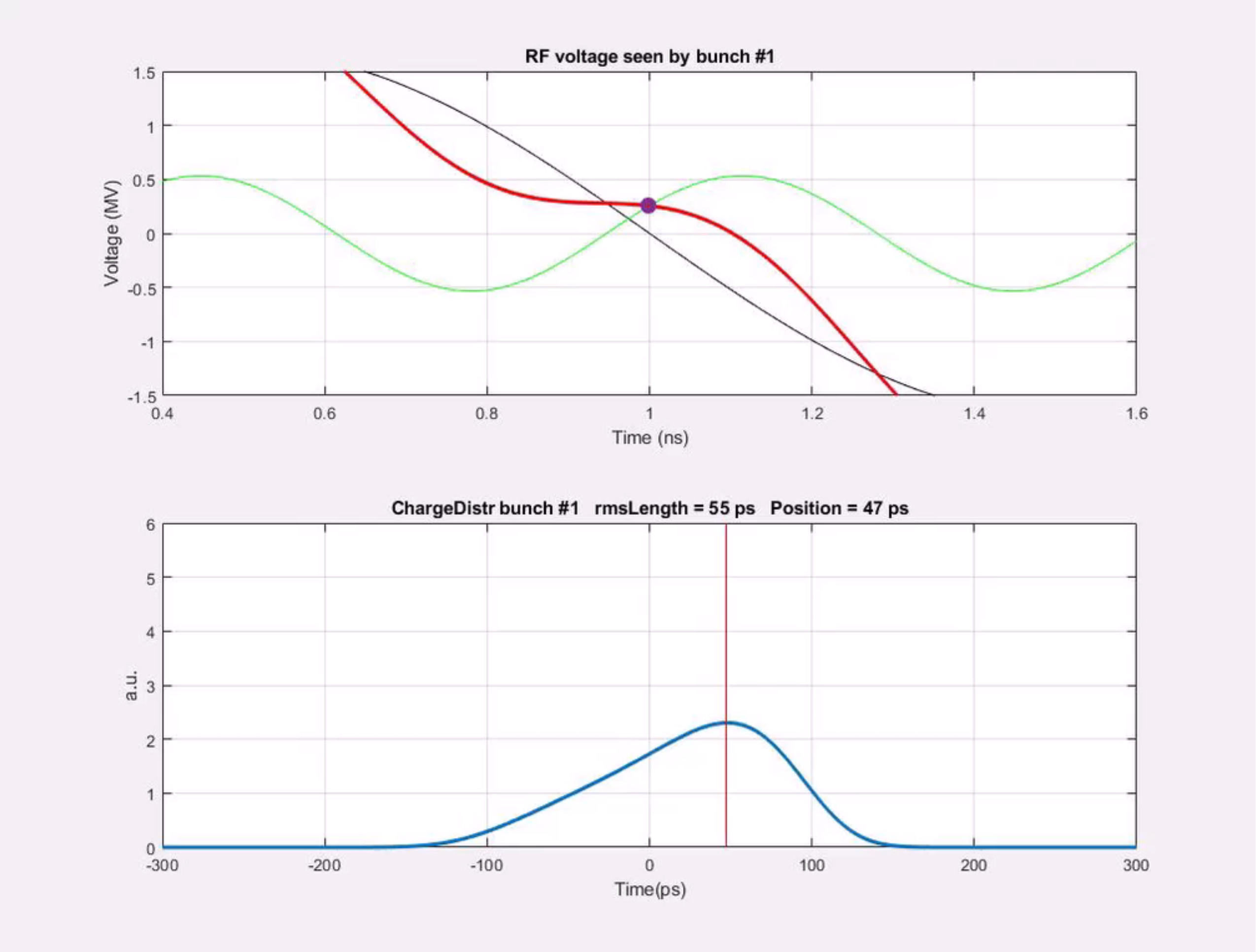
Simulator Results: potential and charge density



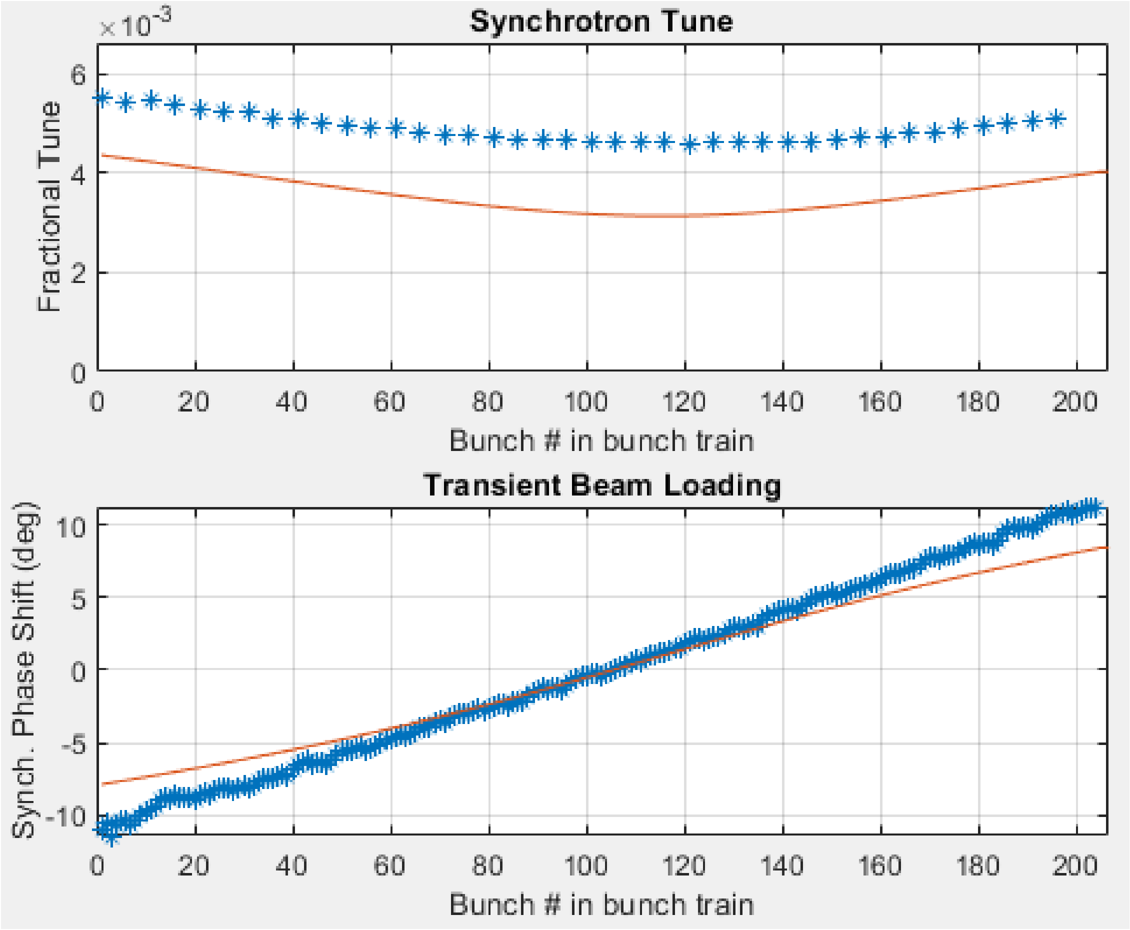
Simulator Results: total voltage and bunch profile



Simulator Results: total voltage and bunch profile (animation)



Comparison with the real machine: preliminary



Considerations and outlook

Pending issues

- ✓ Consistency with the real machine still to be understood: a cross check with other simulation tools (tracking codes) would be interesting
- ✓ Some problems of convergence with harmonic cavity working in overstretching

Further developments

- ✓ Calculate the real beam spectrum and not the theoretical one assuming a gaussian bunch (which is not the case)
- ✓ Include the beam loading of the main RF cavity

Future work

- ✓ Perform simulation for Elettra 2.0
- ✓ Possibly create and test on Elettra conditions similar to Elettra 2.0: ex. every other bunch hybrid filling, …

Contacts, acknowledgments and references

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REFs

- J. M. Byrd, S. De Santis, J. Jacob, and V. Serriere, "*Transient beam loading effects in harmonic rf systems for light sources*", Phys. Rev. ST Accel. Beams 5, 092001 – 2002
- G. Penco and M. Svandrlik, "*Experimental studies on transient beam loading effects in the presence of a superconducting third harmonic cavity*", Phys. Rev. ST Accel. Beams 9, 044401 – 2006

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Thank you!

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