# Topological data analysis in visualizaiton

Ingrid Hotz, LiU Lund, October 3, 2024



Concepts, Methods, Boots, Boot

# Topology Study of shape invariants under continuous deformation



Image: http://math.arizona.edu/~models/Topology/image/coffee\_cup\_to\_donut.jpg

A topologist does not know the difference between a coffee cup and a donut.

Algebraic topology	Computational topology	Topology in Visualiaiton
Concepts, theorems	Algorithms, Practical solutions to topological problems	Semantic abstractions Answer domain specific questions
Classify shapes Investigate spaces via algebraic invariants	Reliable, robust, efficient Strong guarantees $( < \epsilon)$	Mapping to the domain needs

# Topological Descriptors frequently used in Visualizaion



Merge tree

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Persistence diagrams Barcodes



Mores-Smale complex Extremal structures

# Merge Tree

Hydrogen: Electron Density Field *s* Set of nested isosurface

 $\{(x, y, z) \in D | s(x, y, z) = C_i\} = s^{-1}(C_i)$ for a scalar function *s* and iso values  $C_i \in \mathbb{R}, i = 1 \dots n$ 

> Image: Martin Falk, LiU SW: Inviwo

# Merge Tree

Electron Density Field Hydrogen Isosurface representation



# Utilizing the merge tree for data summarization





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#### International collaboration between

- TU Kaiserslautern, Germany
- Linköping University, Sweden



## Electronic structures Utilizing the merge tree for summarization and comparison

[Exploring Electron Density Evolution using Merge Tree Mappings. Wetzels et al, 2024]

Compute 'Edit Distance distance' between two merge trees  $\rightarrow$  induces mappings between edges in the merge trees.





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Edit Distance distance between two merge trees  $T_1$ ,  $T_2$ 

is defined to be the minimal cost of an edit sequence transforming  $T_1$  into  $T_2$  $\delta_E(T_1, T_2) = \min\{c(S) | T_1 \xrightarrow{S} T_2\}.$ 

The deformation edit distances are a metric for (abstract) merge trees



# Correlation with nuclear structure { $p_1, \dots, p_n$ },



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### Visualization interface

- Four distance matrices based on the TED,  $d_1$ ,  $d_2$ ,  $d_3$
- Correlation bars between the values of TED and each of  $d_1$ ,  $d_2$ ,  $d_3$  for selected submatrices.
- Timeline View for nuclear structures
- Merge Tree View.
- Spatial contours evolutions/changes for two selected time-steps



# Analysis of Structures in Solid-State Materials

Signe Sidwall Thygesen, Alexei I. Abrikosov, Peter Steneteg, Talha Bin Masood, Ingrid Hotz

Collaboration: Igor Abrikossov, LiU

[Thygesen et al. 2023. Level of Detail Visual Analysis of Structures in Solid-State Materials] [Bykov et al. 2021. High-Pressure Synthesis of Dirac Materials: Layered van der Waals Bonded BeN4 Polymorph]



# The collaboration and context – visualization challenge

### Data per unit cell

- Charge density for BeN<sub>4</sub> at 7 different pressure points (0 to 83.6 GPa)
- Atomic positions

### **Questions:**

- Can you produce nicer pictures for our publication
- Strengthen the **evidence of layers** based on charge density data?
- Provide "data summary" that facilitates automatic and visual distinction between layered vs unlayered configurations?









### Black and white isosurfaces show the separations of layers briefly before thy merge









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### Morse-Smale complex

Partitioning of the domain in regions of uniform gradient behavior

- Abstraction while maintaining more geometric information
- Spatial segmentation and feature definitions
- Support multi scale analysis



Critical points: green minima, blue saddle, and red maxima Separatrices (black) are gradient lines forming the boundary between lines that flow towards different maxima. Ingrid Hotz,LiU, October 2024

## Morse-Smale complex

Partitioning of the domain in regions of uniform gradient behavior

Scalar field with critical points

Gradient lines

Descending manifolds (DM) partition the domain into mountains

Saddle point (blue) Its AM (bold black line) connects the two maxima.









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Images: Talha Bin Masood

# Project Granular materials Properties and scales

Discrete materials with complex mechanical behavior. Properties akin to solids, fluids, and gases depending on particle arrangement, and particle morphology.

### Material scientists: • Tejas Murthy

### Data analysis and visualization experts

- Talha Bin Masood,
- Vijay Natarajan
- Ingrid Hotz

### International collaboration between

- IISC, Bangalore, India
- Linköping University, Sweden

# X-ray CT experiments

The specimen is placed in front of an X-ray source, mounted on a rotating table.

Projections at small increments of rotation are captured on a detector.



# X-ray CT experiments





# MorseGram: Segmentation and Fabric Quantification of Granular Materials



CT- image



Grain segmentation



**Contact Network** 

[K Pandey, T BinMasood, S Singh, I Hotz, V Natarajan, TG Murthy. Morse theory-based segmentation and fabric quantification of granular materials. Granular Matter, 24:1–20, 2022. ]

# MorseGram: Segmentation and Fabric Quantification of Granular Materials



[K Pandey, T BinMasood, S Singh, I Hotz, V Natarajan, TG Murthy. Morse theory-based segmentation and fabric quantification of granular materials. Granular Matter, 24:1–20, 2022. ]

# MorseGram - pipeline

- Bounding surface computation based on an automatically determined bi-modal threshold and a local active contourbased refinement.
- Signed distance field-computation
- MS complex extraction for distance field (multi-scale).
- Persistence curve used to determine simplification threshold
- Segmentation is computed and reported as the descending manifolds of the maxima in the simplified complex.
- The connectivity network is computed as the collection of ascending 1-manifolds of 2-saddles and the contact region is computed by pruning the descending 2-manifold of the 2-saddles



# MorseGramVis – Open-Source Software



[D Rathod. MorseGramVis: A visualization and analysis tool for segmented granular media. Masters thesis, Indian Institute of Science Bangalore, Department of Computer Science and Automation, 2023]