Some “things” we want to know

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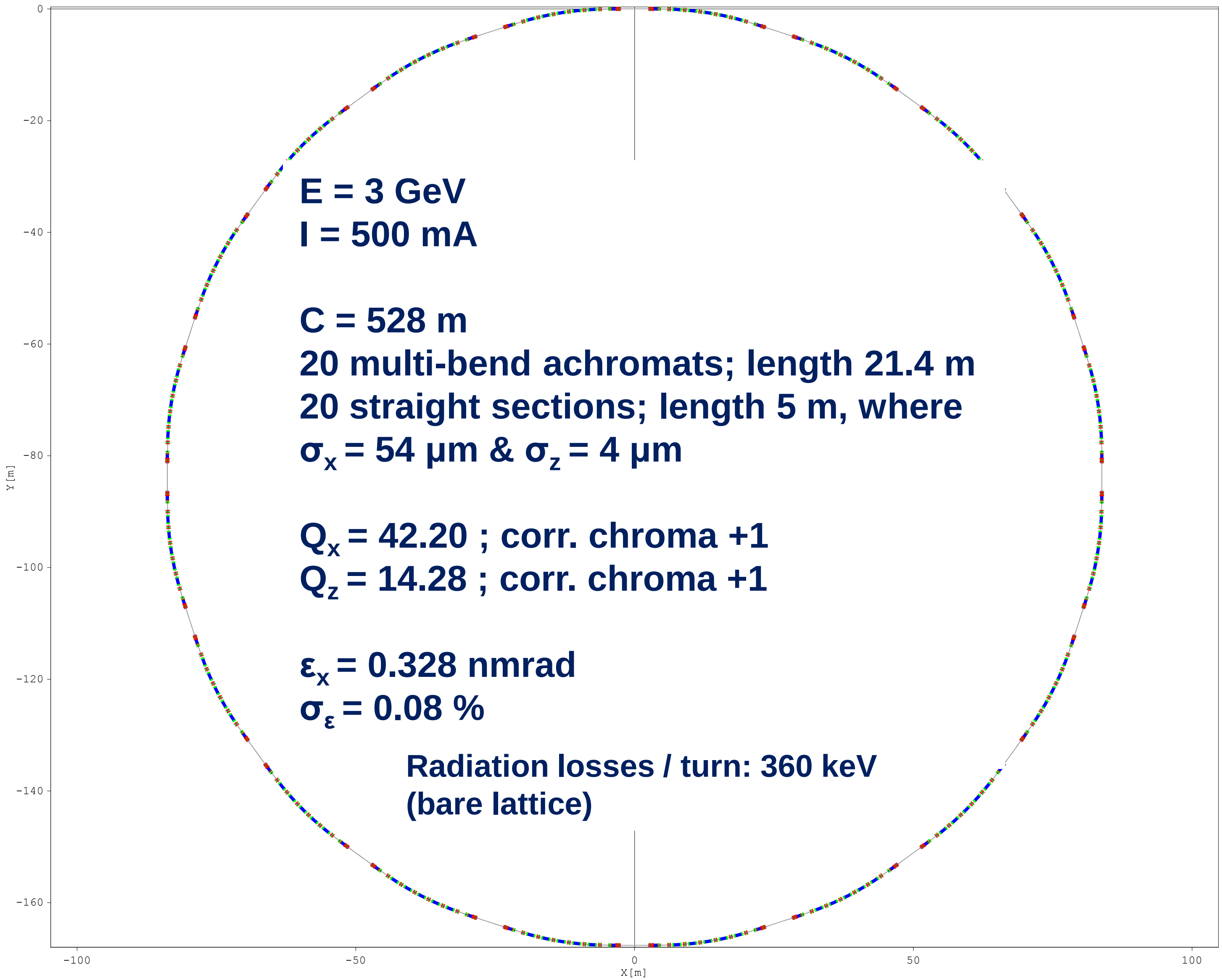
Accelerator Development Group

Outlook

Abstract:

We will try to list some properties of an accelerator, that is of high interest to know. We will then encounter some concepts commonly used in the control room discussions, and I will try to explain them. The presentation will be skewed towards ring accelerators like our storage rings.

MAX IV 3 GeV Storage Ring



E = 3 GeV
I = 500 mA

C = 528 m
20 multi-bend achromats; length 21.4 m
20 straight sections; length 5 m, where
σ_x = 54 µm & σ_z = 4 µm

Q_x = 42.20 ; corr. chroma +1
Q_z = 14.28 ; corr. chroma +1

ε_x = 0.328 nmrad
σ_ε = 0.08 %

Radiation losses / turn: 360 keV
(bare lattice)

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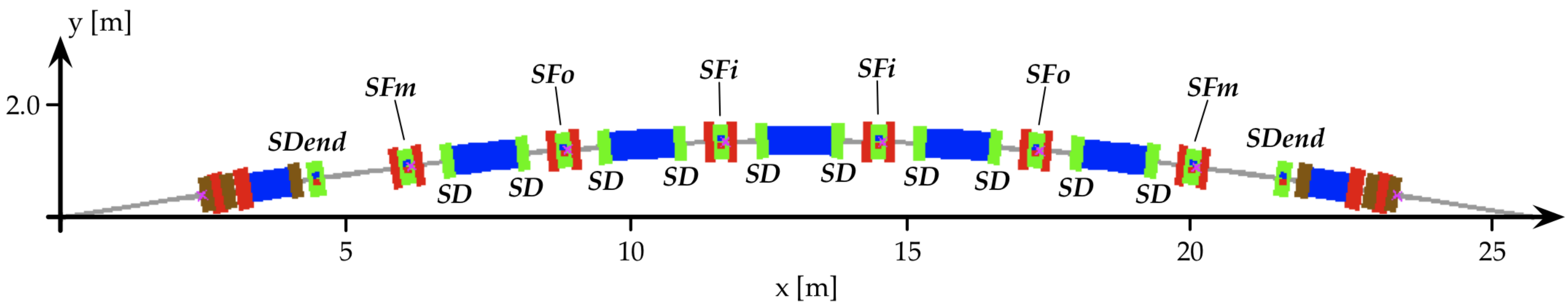
Outlook

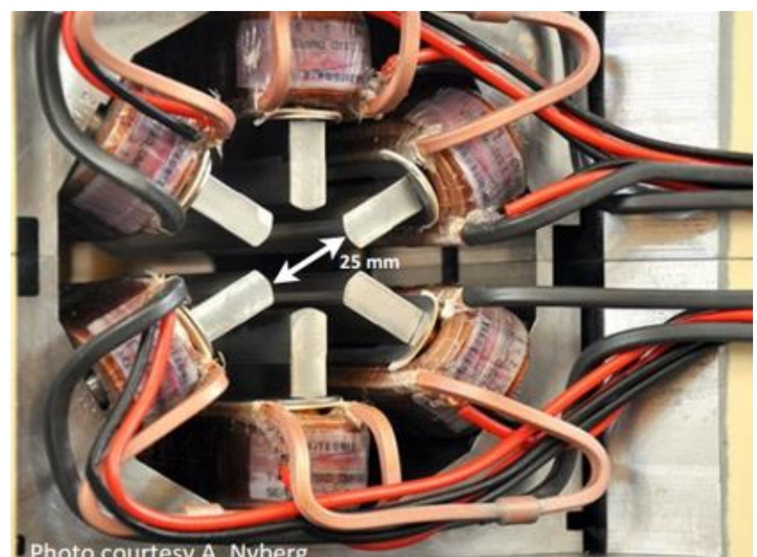
Some ”things” we’d like to know about our storage ring:

- The beam position
- The beam focusing (quadrupole magnet strengths)
- The beam focusing corrections (sextupole magnet strengths)
- The longitudinal focusing of the beam (cavity voltages)
- The beam intensity (“The current”)
- The beam energy
- The beam energy spread and the bunch lengths
- The beam size and angular spread in hor. and vert. dimension
- The dominant electron loss mechanisms (max deviation in position/angle or energy)

MAX IV 3 GeV ring



- Bending (or dipole) magnets
- Quadrupole magnets
- Sextupole magnets
- Octupole magnets

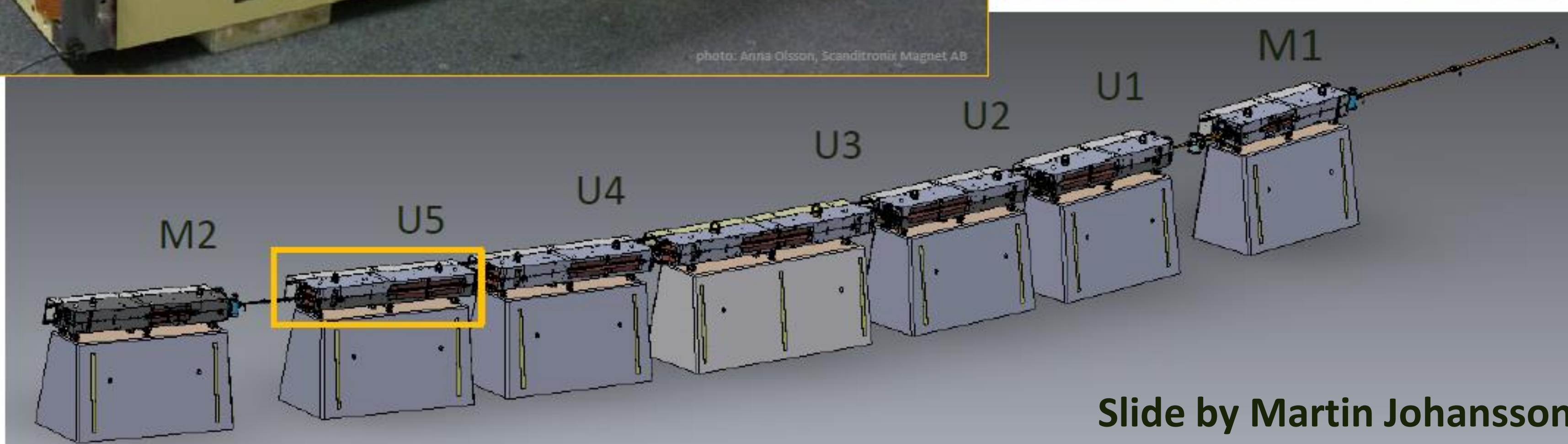


MAX IV 3 GeV ring DC magnets

• Each cell is realized as one mechanical unit containing all magnet elements.

•Each unit consists of a bottom and a top yoke half, machined out of one solid iron block, 2.3-3.4 m long.

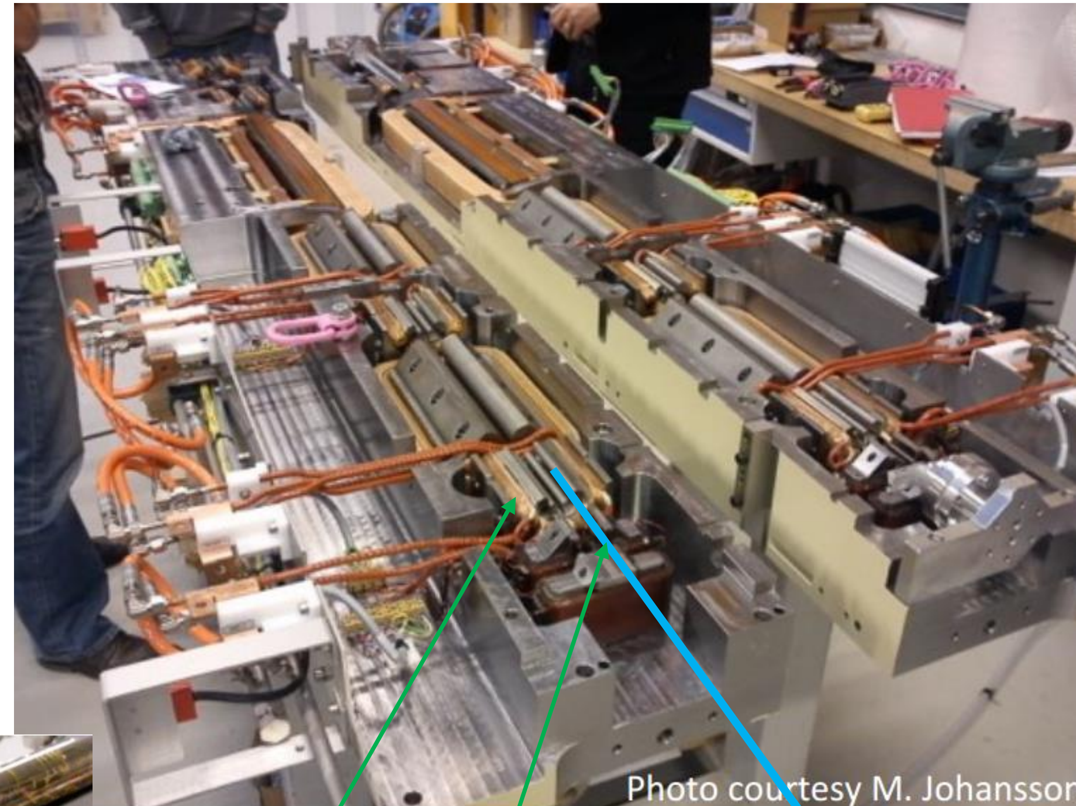
- a U5 bottom half →
- ↓ an assembled U5

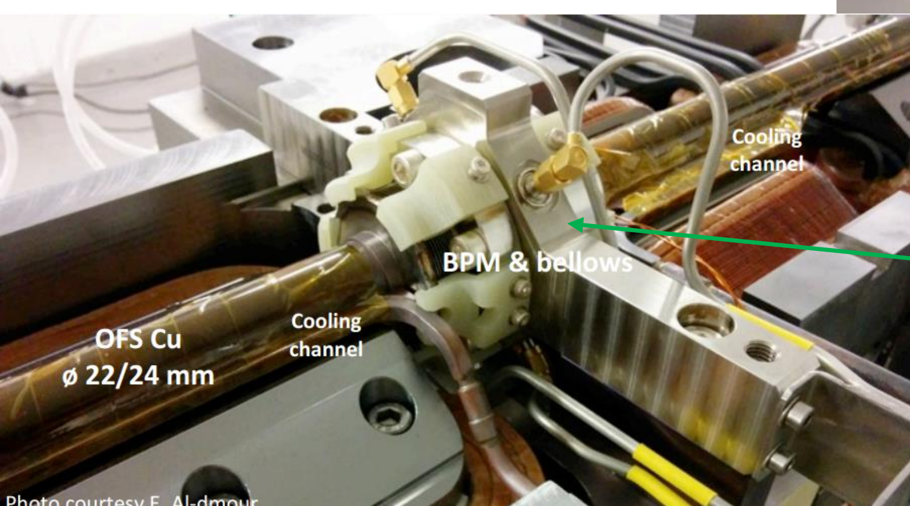


Slide by Martin Johansson

The beam position

- Don’t require much accelerator theory ☺.
- Most important for the SR users, are the positions around the beam line source point, i.e the Insertion Device (ID).





Octupole

Beam Position Monitor (BPM)

E-beam through ID

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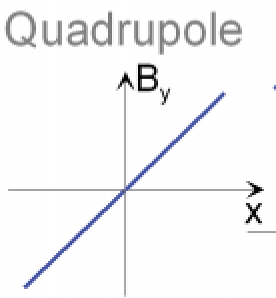
The beam position

- The beam should be positioned to within one tenth of the hor/vert beam size (54um/4um). The BPMs have this relative (short term) precision, but the absolute position must be repeatedly verified.
- In the surrounding octupoles we have auxiliary windings (Trim Coils) on all poles.
- We create a weak quadrupolar field by means of the trim coils. The beam can only pass unaffected through this field, if it passes in the so-called magnetic centre. The absolute ”zero position” is this centre.
- Beam Based Calibration (BBC) , is the procedure by which we calibrate the nearby BPM to report (x,z) = (0,0) for exactly this beam position.
- Sloppy expression: ”Offsets measurements”.
- It takes up to 16 hours to calibrate all 200 BPMs, verifying the zero position to better than a few microns. Refined and faster methods to come….



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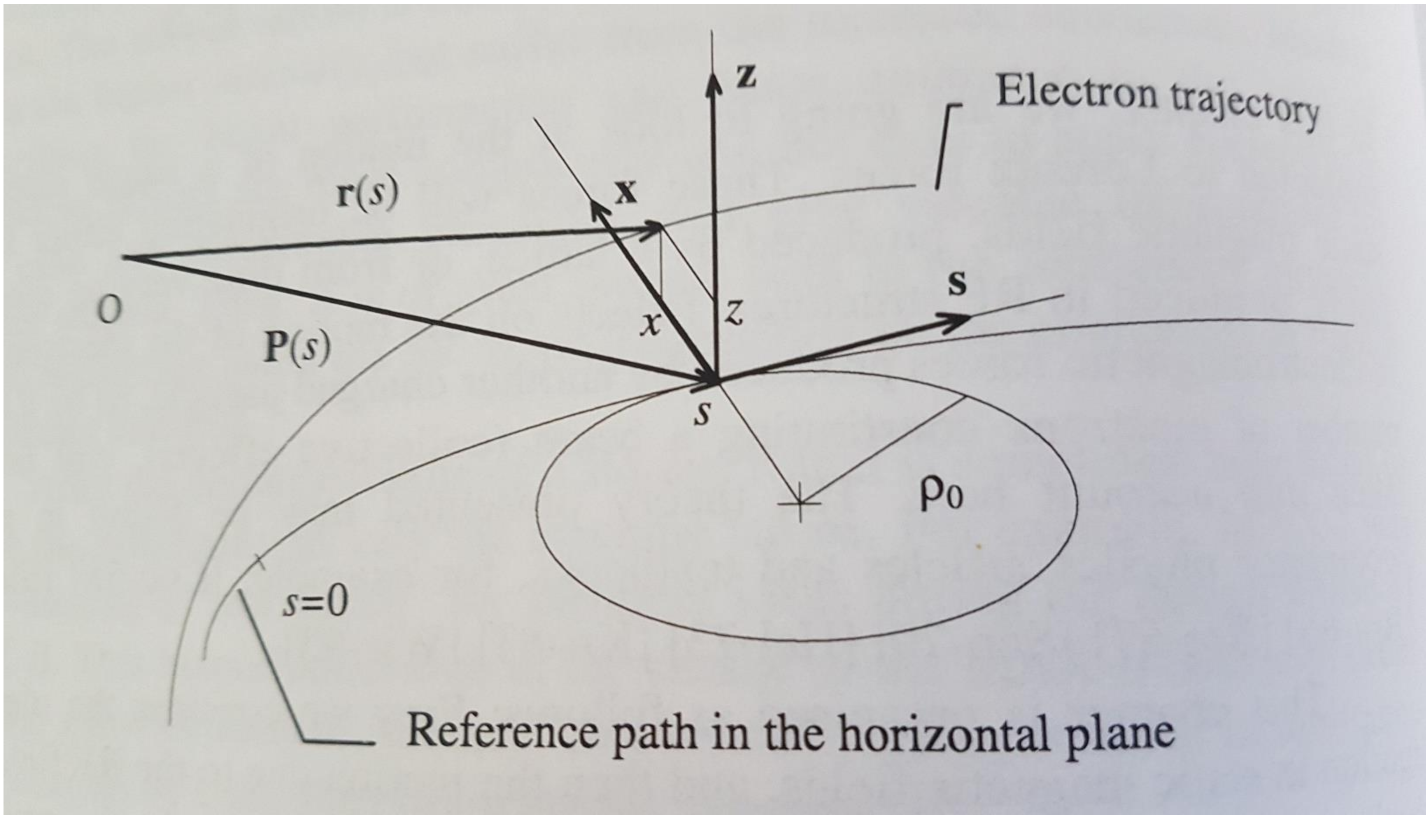
The beam focusing

- Here is needed some more theory:

A moving co-ordinate system



Imagine that we follow a reference particle around the storage ring. Let's calculate how **another** particle moves in x and z directions, when it is not perfectly on the reference orbit:

Magnet strengths

The movement will be dependent on the magnet strengths, given by:

The coefficients (dipole, quadrupole, sextupole) are given by

$$h = \frac{e}{p_0} B_z(0,0,s) = 1/\rho_0(s)$$

$$k = \frac{e}{p_0} \frac{\partial B_z}{\partial x}\bigg|_{x=z=0}$$

$$m = \frac{e}{p_0} \frac{\partial^2 B_z}{\partial x^2}\bigg|_{x=z=0}$$

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Equations of motion

$x' = dx/ds$

with $\delta = (p - p_0)/p_0$, being the relative momentum deviation

$$x'' + (h^2 + k)x = h\delta - \left(2hk + h^3 + \tfrac{1}{2}m\right)x^2 + h'xx' + \tfrac{1}{2}hx'^2 + (k + 2h^2)x\delta + \tfrac{1}{2}(h'' + hk + m)z^2 + h'zz' - \tfrac{1}{2}hz'^2 - h\delta^2$$

and

$$z'' - kz = (2hk + m)xz + h'xz' + h'x'z + hx'z' + kx\delta$$

Needs simplification!

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Eq. of motion to first order, at p_0

$$x''(s) + (h^2(s) + k(s))x(s) = 0$$

$$z''(s) - k(s)z(s) = 0$$

constraints for a storage ring

$$h(s) = h(s + C_0) \qquad k(s) = k(s + C_0)$$

Similar to:

$$x''(t) + \left(\frac{k}{m}\right)x(t) = 0$$

Solution:

$$x(t) = A\sin(\omega t + \varphi), \omega = \sqrt{k/m}$$

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Hill differential equation

$$x''(s) + (h^2(s) + k(s))x(s) = 0$$

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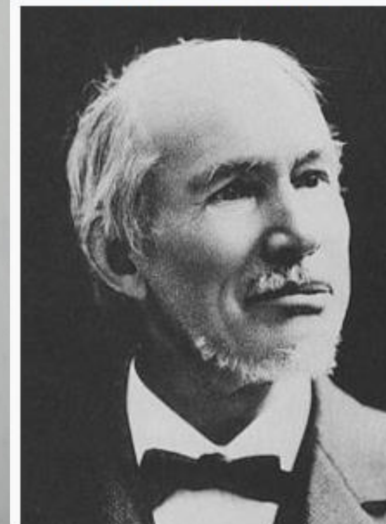
George William Hill



George William Hill

US astronomer, 1838-1914

Solution:

$$x(s) = A\sqrt{\beta_x(s)}sin(\Psi_x(s) + \Psi_0)\ , \qquad where\ \Psi_x(s) = \int_0^s \frac{d\hat{s}}{\beta(\hat{s})}$$

Hill differential equation

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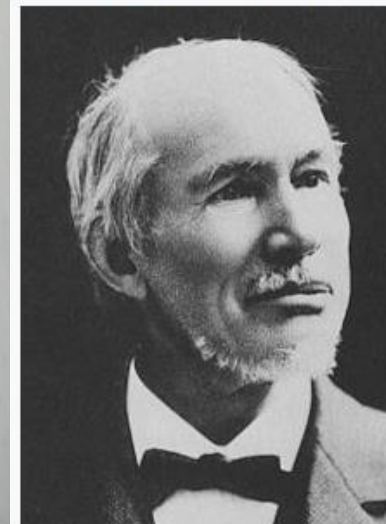

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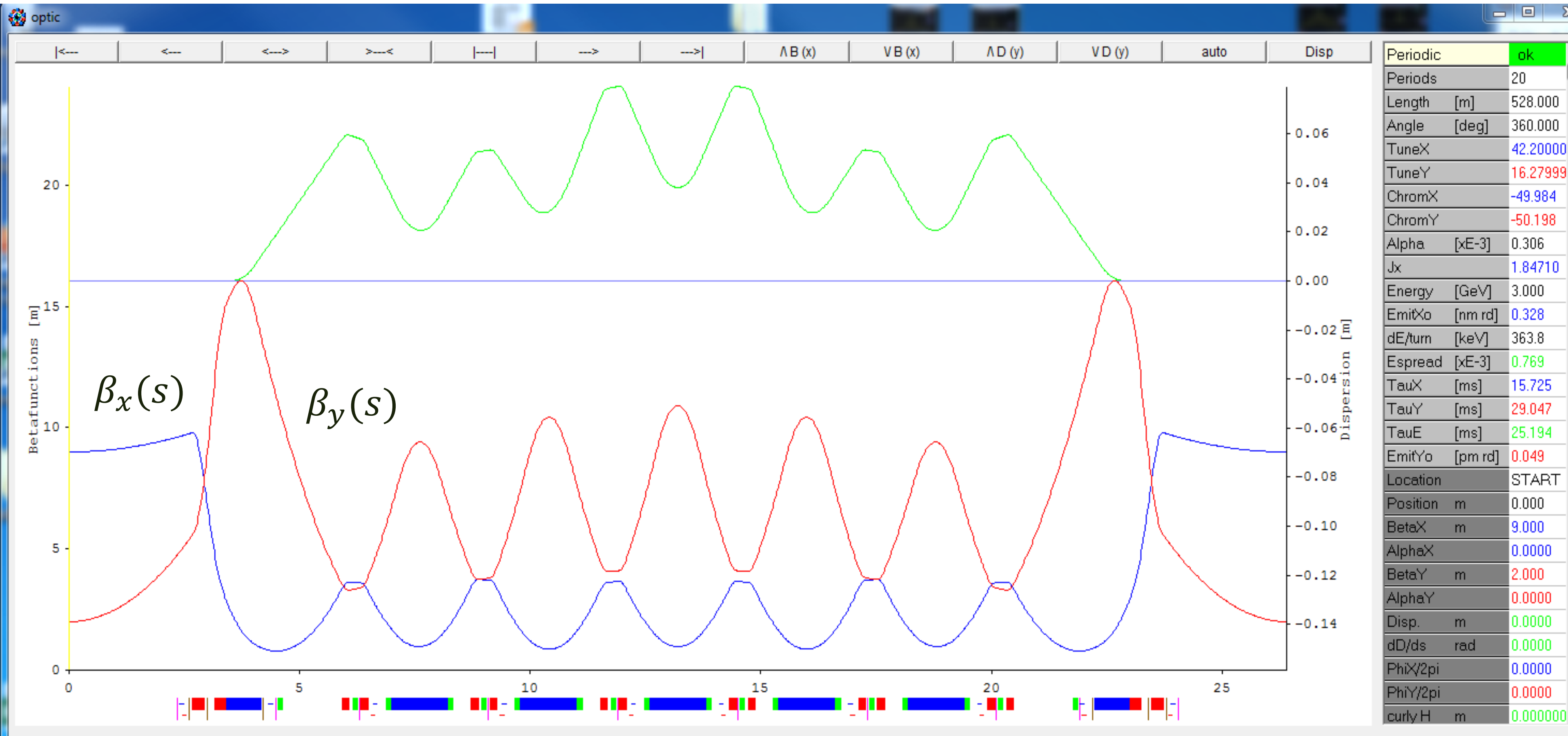
Solution:

$$especially\ Q_x = \frac{1}{2\pi}\int_0^{C_0} \frac{d\hat{s}}{\beta(\hat{s})}$$

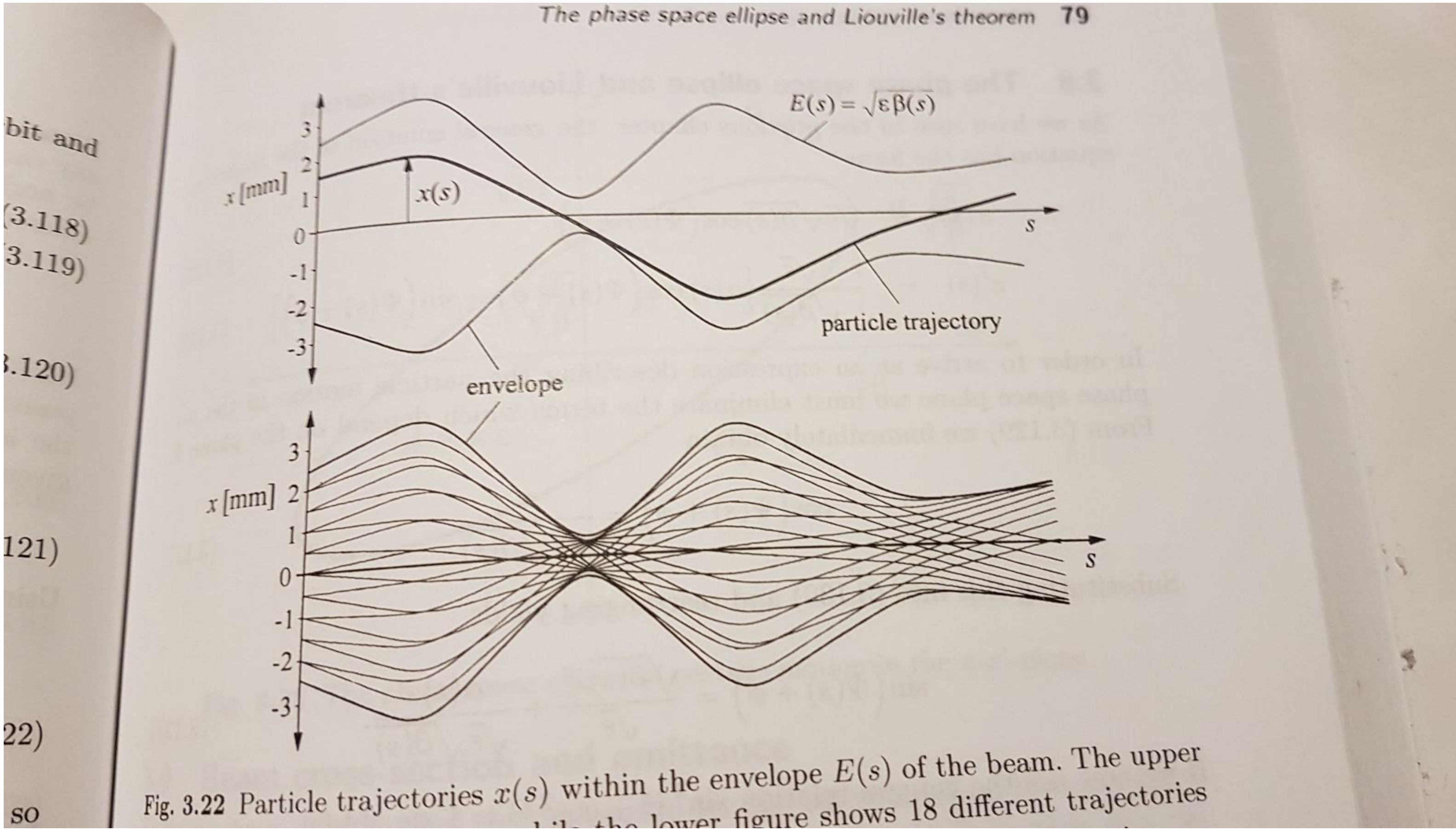
Hill differential equation

β(s) is also periodic , $\beta(s) = \beta(s + C_0)$, and fullfils: $\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta' + \beta^2(h^2 + k) = 1$

This is the ”famous” beta-function, that we try to measure in the control room.



Particle envelop



One electron will turn after turn pass the same s-value with different Ψ, $\Psi(s) \neq \Psi(s + C_0)$. It will "track out" an envelop $E(s) = A\sqrt{\beta(s)}$

Courtesy K. Wille, The physics of particle accelerators

Horizontal beam envelop, or beam size

The horizontal rms beam size is given by:

$$\sigma_x(s) = \sqrt{\varepsilon_x \beta(s) + \sigma_\varepsilon^2 \eta^2(s)}$$

Equilibrium horizontal beam emittance

Equilibrium beam energy spread

These equilibria are reached within tens of milliseconds after a beam is injected, or after the stored beam was disturbed. It is a feature of electron (/positron) storage rings, where the quantum nature of photon emission rules.

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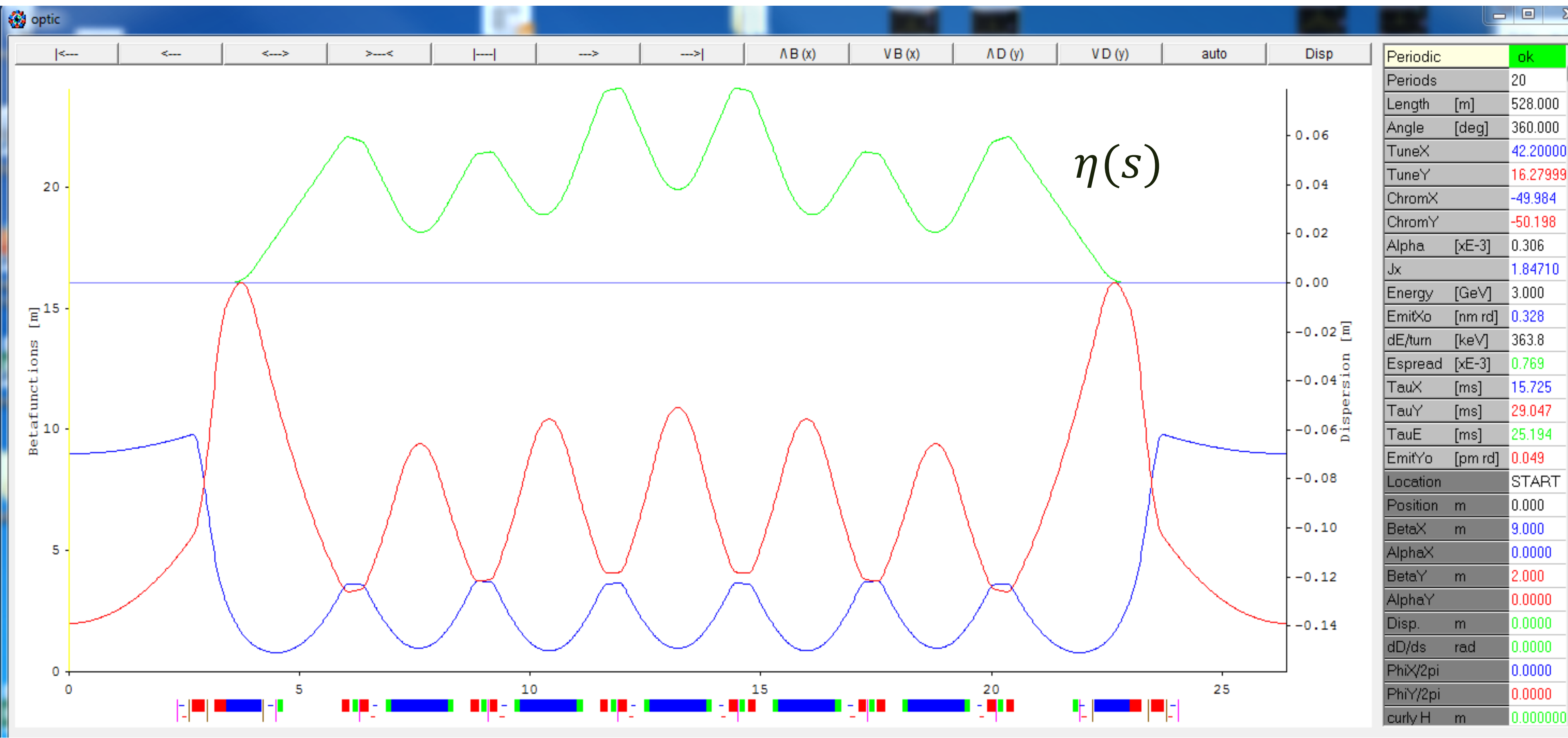
Dispersion function

The dispersion function η(s), is the periodic solution to the inhomogenous equ:

$$\eta''(s) + (h^2(s) + k(s))\eta(s) = h(s)$$

$\eta(s)\delta$ Describes the closed orbit for a particle with a relative momentum deviation $\delta = (p - p_0)/p_0$



Important message

If we can verify/measure k(s), h(s), $\beta_x(s)$, $\eta(s)$ both the horizontal beam emittance and the beam energy spread are given through the so-called synchrotron radiation integrals.

EVALUATION OF SYNCHROTRON RADIATION INTEGRALS*

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Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

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University of California, Santa Cruz, California 95060

The Integrals

We restrict our attention to guide fields made up of a number of magnetic segments — magnets or straight sections. The functions ρ and n are assumed to have constant values within a given magnet, but vary abruptly at the entrance and exit boundaries. The integrals of interest are given by:

$$I_1 = \oint (\eta/\rho) ds = \sum_i \frac{\ell_i}{\rho_i} \langle \eta \rangle_i \qquad (1)$$

$$I_2 = \oint (1/\rho^2) ds = \sum_i \frac{\ell_i}{\rho_i^2} \qquad (2)$$

$$I_3 = \oint |1/\rho|^3 ds = \sum_i \frac{\ell_i}{|\rho_i|^3} \qquad (3)$$

$$I_4 = \oint \frac{(1-2n)\eta}{\rho^3} ds = \sum_i \left[\frac{\ell_i}{\rho_i^3} \langle \eta \rangle_i - 2\ell_i \left\langle \frac{n\eta}{\rho^3} \right\rangle_i \right] \qquad (4)$$

$$I_5 = \oint \frac{H}{|\rho|^3} ds = \sum_i \frac{\ell_i}{|\rho_i|^3} \langle H \rangle_i \qquad (5)$$

We have used the notation $\langle f \rangle_i$ for the mean value of f in the i^{th} segment whose length is ℓ_i. The function H(s) is defined by

$$H = \frac{1}{\beta} \left\{ \eta^2 + (\beta\eta' - \frac{1}{2}\beta'\eta)^2 \right\} \qquad (6)$$

with $\beta' = d\beta/ds$, and $\eta' = d\eta/ds$. It should be noted right away that at least one factor of $1/\rho$ appears in each integral; so the straight sections or pure quadrupoles make no contribution.

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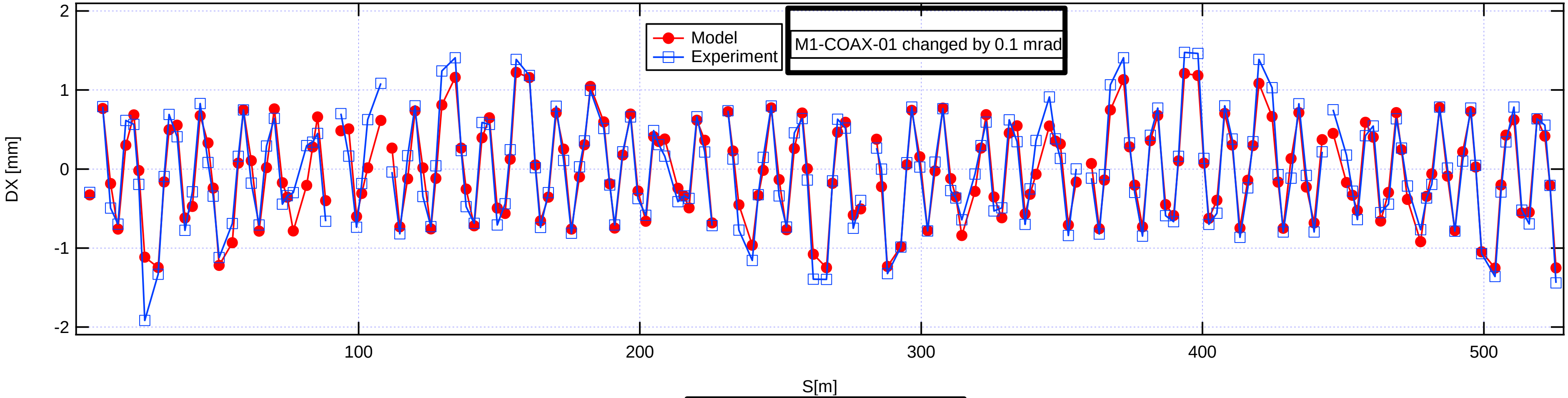
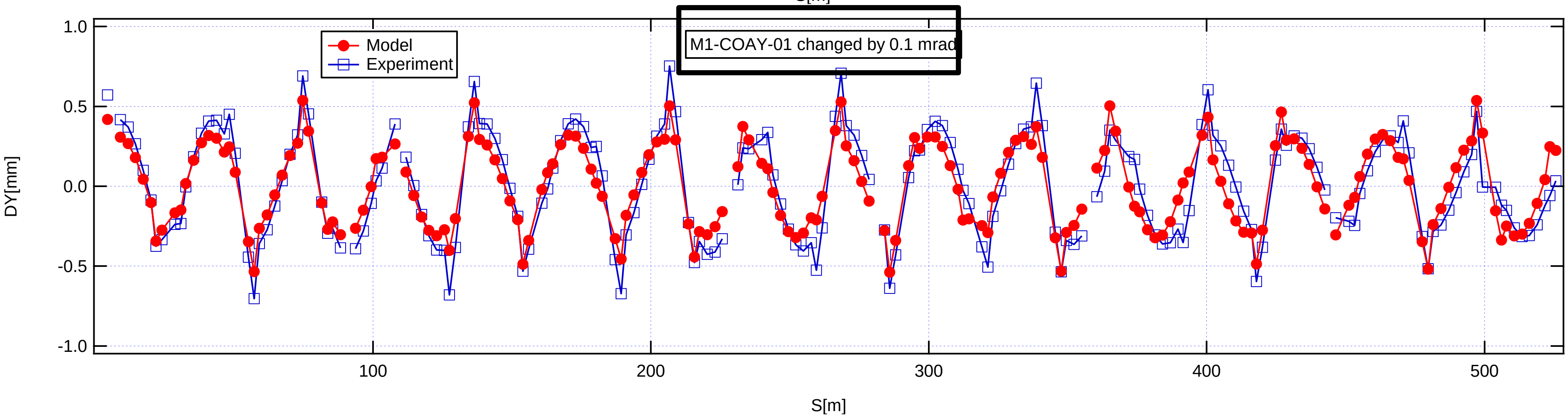
Will be known at low current in the ring!

Measuring the beta- and dispersion functions

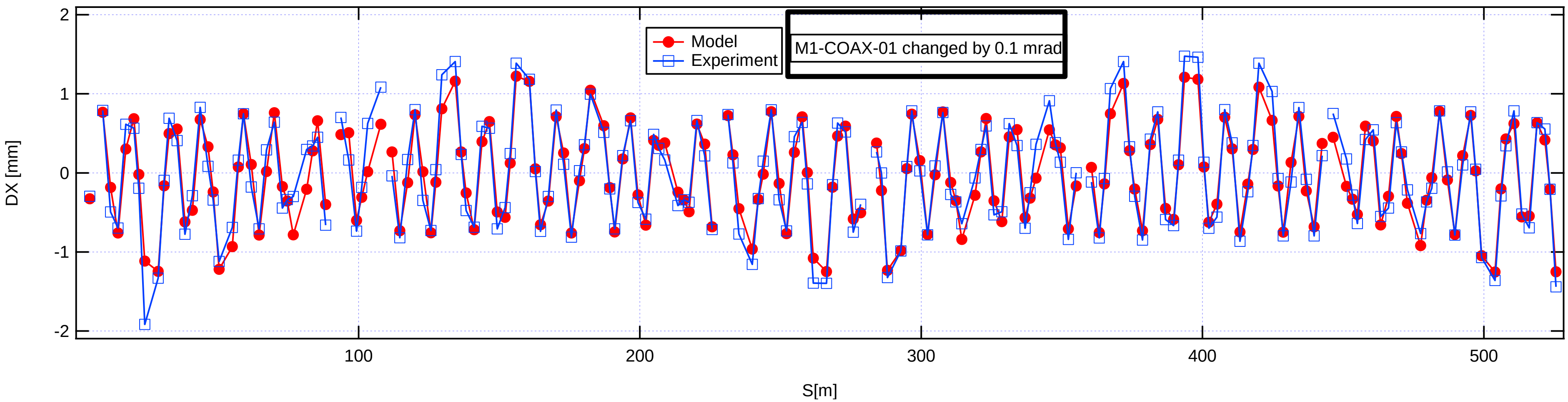
- We add a tiny dipole field by help of a ”corrector” magnet
- Observe the new orbit in the ring, at 200 locations (BPMs).
- We do it with 200 different correctors horizontally.
- We do it with 190 different correctors vertically.
- Finally we change the beam energy up and down, by a known amount (typically with a few tenths of a %)
- Observe the two orbits, at 200 BPMs.
- The horizontal positions give the dispersion functions (at BPMs).
- The above procedure is called Orbit Response Matrix measurement, and takes about one hour.

Measuring the beta- and dispersion functions



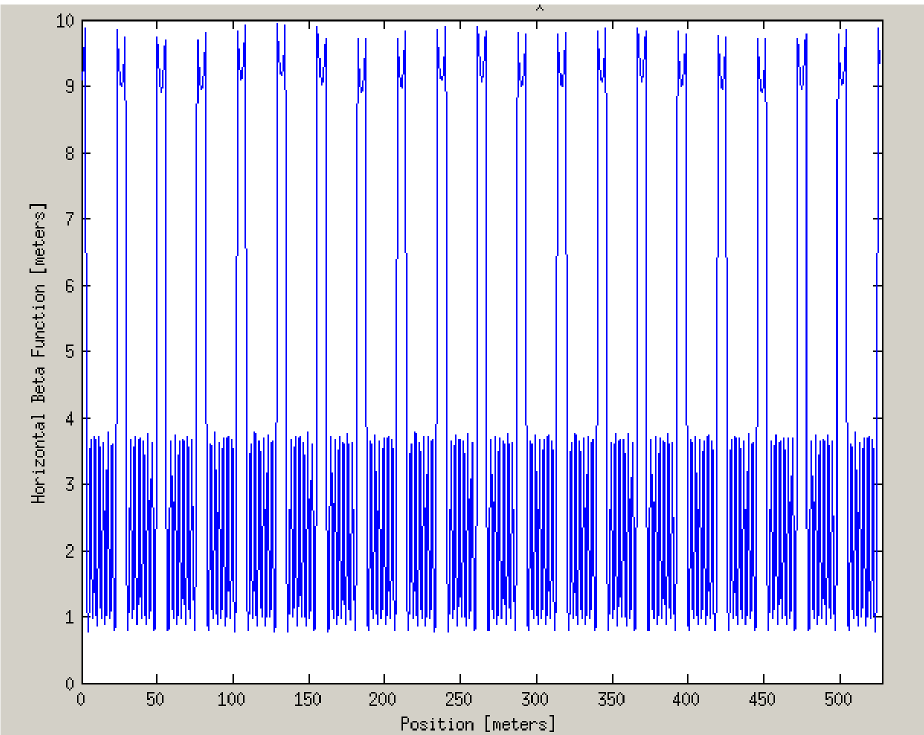


Measuring the beta- and dispersion functions

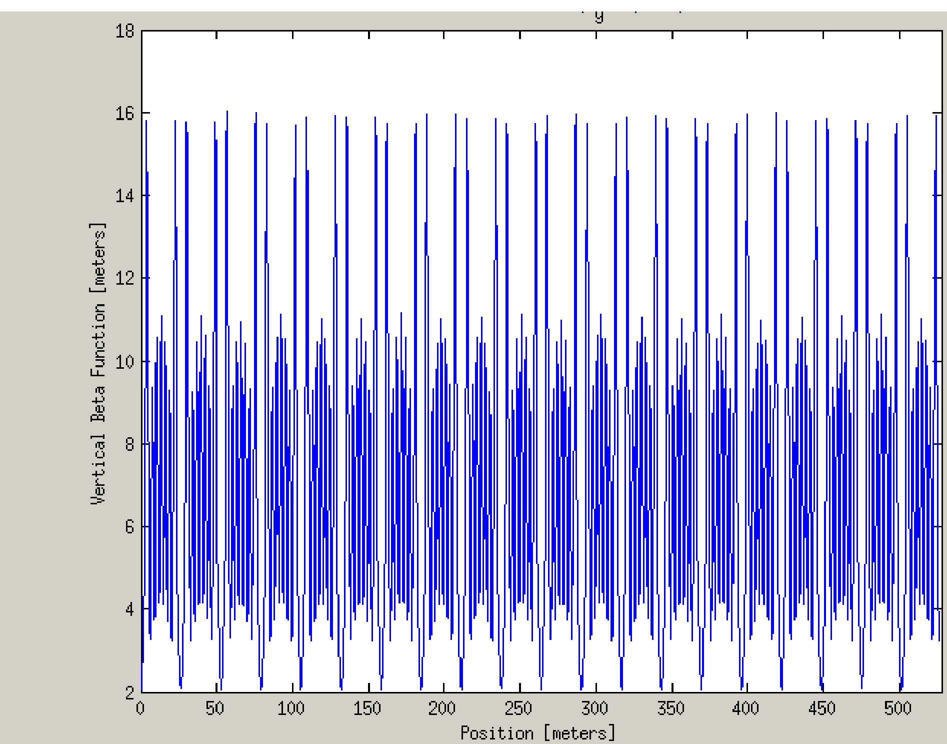


$$x_c(s) = \frac{x'_k}{2\sin(\pi Q_x)}\sqrt{\beta_x(s_k)}cos(\Psi_x(s) - \pi Q_x) \ \ , where \ \Psi_x(s) = \int_0^s \frac{d\hat{s}}{\beta(\hat{s})}$$

Measuring the beta- and dispersion functions

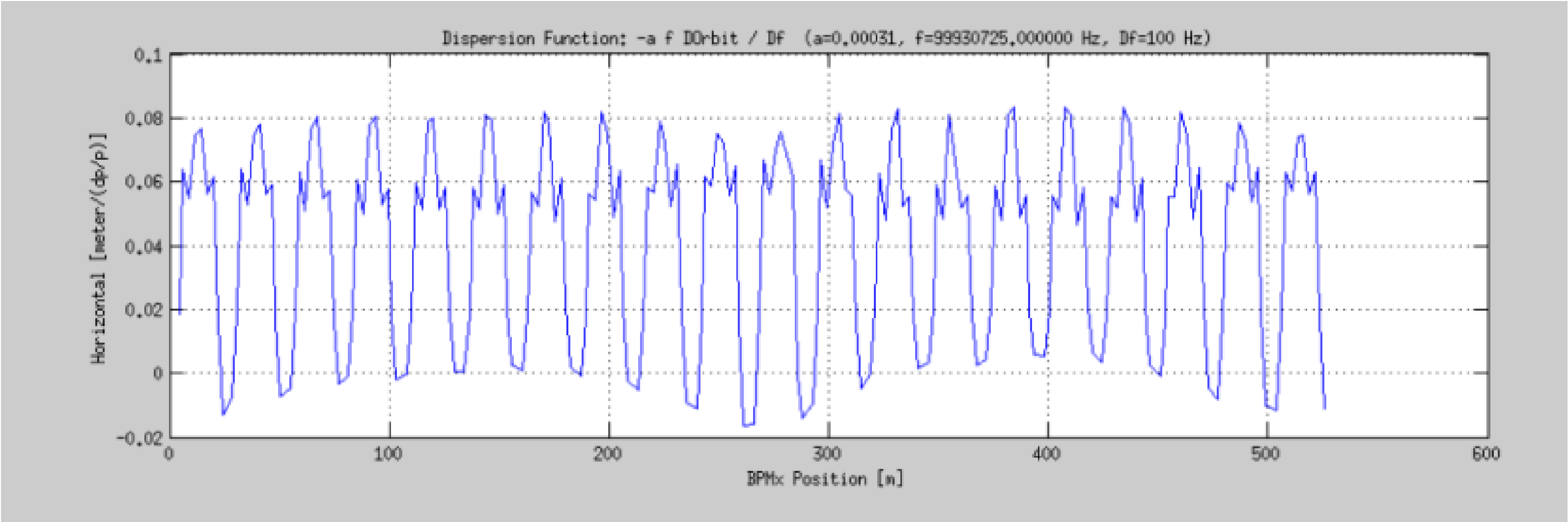


Corrected Horizontal Beta-function



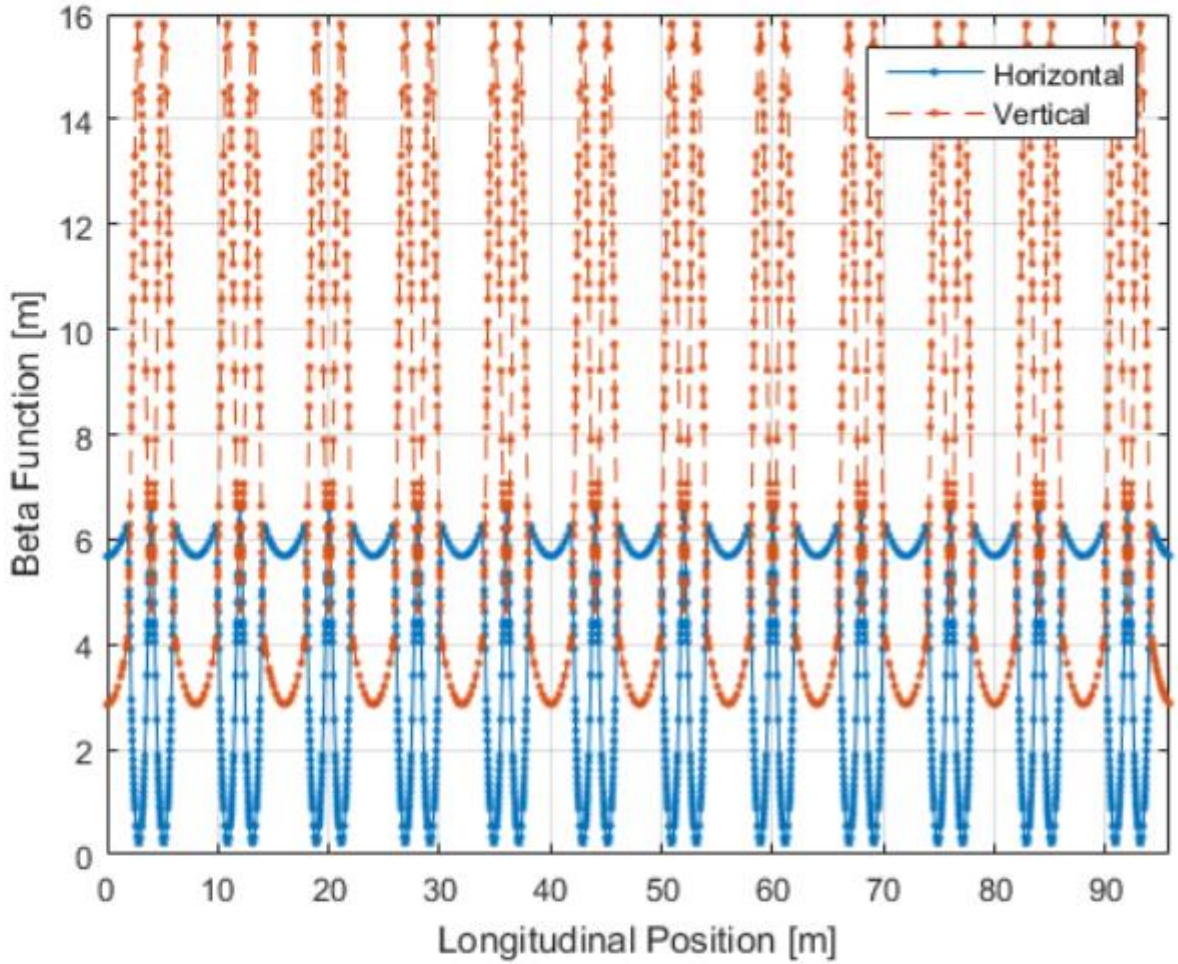
Corrected Vertical Beta-function

Measuring the beta- and dispersion functions

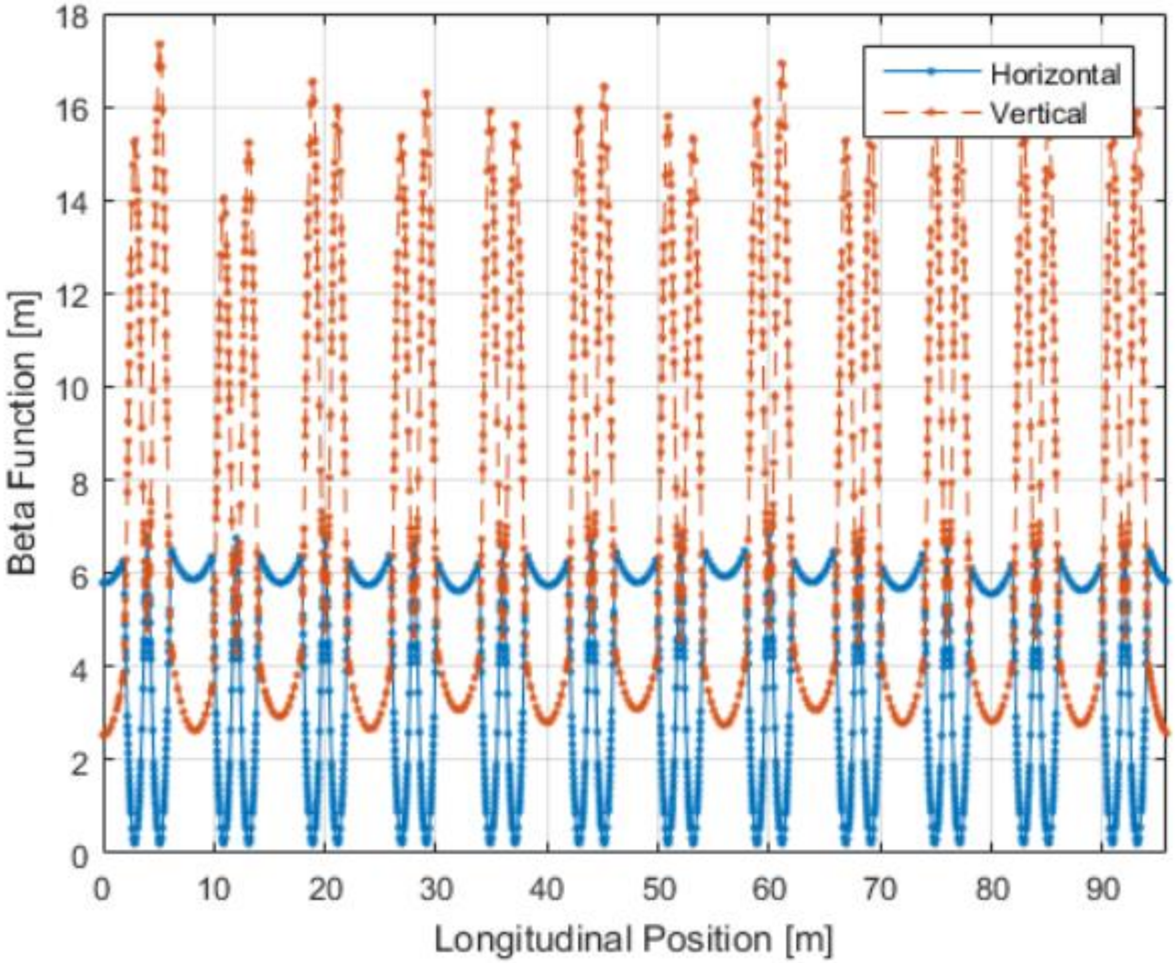


Dispersion function, uncorrected

1.5 GeV Ring Linear Optics Design



1.5 GeV Ring Linear Optics Reality



Horizontal $\frac{\Delta\beta}{\beta} < 5\%$

Vertical $\frac{\Delta\beta}{\beta} < 12\%$

(spring 2017)

Magnets are as from supplier. No shunts are yet applied.

Data by David K. Olsson

Verifying horizontal beam emittance

The horizontal rms beam size is given by:

$$\sigma_x(s) = \sqrt{\varepsilon_x \beta(s) + \sigma_\varepsilon^2 \eta^2(s)}$$

Equilibrium horizontal beam emittance

Equilibrium beam energy spread

- At high current in the ring, we will have new equilibriums:
- However, we trust the measured beta and dispersion functions
- Measure at two locations the beam sizes, and

$$\mathbb{E}_x = \frac{\sigma_{x,2}^2 - \left(\frac{\eta_{x,2}}{\eta_{x,1}}\right)^2 \sigma_{x,1}^2}{\beta_{x,2} - \left(\frac{\eta_{x,2}}{\eta_{x,1}}\right)^2 \beta_{x,1}}$$

$$\sigma_\delta = \left[\frac{\sigma_{x,2}^2 - \left(\frac{\beta_{x,2}}{\beta_{x,1}}\right)\sigma_{x,1}^2}{\eta_{x,2}^2 - \left(\frac{\beta_{x,2}}{\beta_{x,1}}\right)\eta_{x,1}^2}\right]^{1/2}$$

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Sorry, hopefully I can sort out red areas in a second seminar.
THANK YOU!!